Exam Stochastic Processes

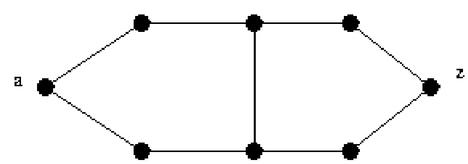
June 2020

- You have from 15.00 until 18.30. This includes the time needed to take pictures of your work and upload it to nestor dropbox.
- It is absolutely not allowed to use calculators, phones, computers, books, notes, the help of others or any other aids.

Exercise 1 (20 pts)
Consider the Galton-Watson process with offspring distribution given by $\mathbb{P}(X = k) = (1-p)p^k$ for $k = 0, 1, 2, \ldots$. Give the probability of extinction.
(Plus a derivation; clearly justify all your steps.)

Exercise 2 (20 pts)
State and prove the Paley-Zygmund inequality.
(You may use without proof the Cauchy-Schwartz inequality for random variables.)

Exercise 3 (20 pts)
Consider the following network, where all conductances are one and a, z are as indicated.



Determine the effective resistance and the escape probability.
(Make sure to clearly justify all your steps.)

Exercise 4 (20 pts)
Consider a Markov chain whose transition matrix is given by

$$P = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 \end{pmatrix}.$$

Does it have a unique invariant distribution? (Justify your answer.)
If X_t denotes the state at time t, what can you say about $\mathbb{P}(X_{t+1} = X_t)$ as $t \to \infty$?

Exercise 5 (20 pts)
Consider standard Brownian motion $(B(t))_{t\geq 0}$ and let $a < 0 < b$ be fixed. We define $T := \inf\{t \geq 0 : B(t) \in \{a, b\}\}$. So T is the first time you visit one of a and b. Show that the probability that you reach a before b equals

$$\mathbb{P}(B(T) = a) = \frac{b}{|a| + b}.$$

(*Hint:* You may first want to take a, b rational and consider a comparison with the discrete symmetric random walk, appropriately embedded in B.)

the end